Listy Biometryczne - Biometrical Letters
Vol.28 (1991), No.1, 41-44

THE PROPOSITION FOR TESTING THE SIGNIFICANCE OF COMPETITION COEFFICIENT

ANITA DOBEK

Department of Mathematical and Statistical Methods,
Academy of Agriculture, Wojska Polskiego 28, 60-789 Poznań

Summary

The analysis of an experiment which takes into account the inter-varietal competition is presented. The proposition of testing the significance of this phenomenon as well as the equality of competition coefficients for a series of experiments is described.

1. INTRODUCTION

It is recognized that the variety comparisons may be affected by some uncontrolled effects. One of them is the soil fertility, the other can be the competition between varieties (see Dobek and Kiełczewska, 1987). The problem of taking into account the soil fertility in the analysis of experimental data was widely studied by several authors (for the references see Dagnelie, 1987). The question of incorporating overlap effects from neighbouring treatments was considered among others by Pearce (1957), Mead (1967), Draper and Guttman (1980), Kempton (1982), Besag and Kempton (1986). The overlap effects may be caused by two different sources, namely the inter-varietal competition, specially in small plots where the yield of a variety may be depressed by a more aggressive neighbour and the interference of treatments on neighbouring plots for example when the plots receive different chemical treatment and the wind drift occurs.

If we provide a series of experiments it may be interesting to know if the described above phenomenon is present in each environment (place or year) and if it is the case whether there is a connection between competition coefficients and the environments or if it is a

Key words: inter-varietal competition, maximum likelihood, series of experiments

characteristic of the analysed species. The way in which it is possible to answer this question will be described in what follows.

2. ESTIMATION OF COMPETITION COEFFICIENT

Let us consider a model for a single experiment in which the presence of competition is taken into account, i.e.

$$y = D\beta + \Delta\tau + \alpha W y + e \quad ,$$

where y is the n-vector of observations, D is a nxb design matrix for blocks, β is a b-vector of block effects, Δ is a nxv design matrix for treatments, τ is a v-vector of treatment effects, α is a competition coefficient, W is an off-diagonal matrix with elements $(i,i\pm1)$ equal 1/2 and the others equal to zero and e is n-vector of errors. We assume that the vector of errors has the n-variate normal distribution with $E(e) = 0$ and $Var(e) = \sigma^2 I$.

It should be noted that in this model τ represents the effect of treatments under the competition effect. To obtain the pure treatment effect, as in the monoculture, we should divide τ by $1-\alpha$.

Let us denote by $X = [D,\Delta]$ and $\vartheta = [\beta',\tau']'$. Then the least squares estimates of ϑ and σ^2 are

$$\hat{\vartheta} = (X'X)^{-1}X'Gy \quad , \tag{1}$$

$$\hat{\sigma}^2 = (Gy - X\hat{\vartheta})'(Gy - X\hat{\vartheta})/(n-b-v+1) \quad , \tag{2}$$

where $G = I - \alpha W$.

The conditional maximum likelihood for y is

$$L(y|\hat{\vartheta}, \hat{\sigma}^2) = (2\pi)^{-n/2}|G|\,(\hat{\sigma}^2)^{-n/2}\exp[-(n-b-v+1)/2] \quad .$$

The estimator for α we obtain by maximizing the log-likelihood function

$$l = \ln L = \ln|G| - (n/2)\ln[(Gy - X\hat{\vartheta})'(Gy - X\hat{\vartheta})] \quad .$$

Now it may be interesting whether the coefficient of competition is significant or not. To test the significance of α we calculate additionally $l_o = \ln L_o$ where

$$L_o = L(y|\hat{\vartheta}, \hat{\sigma}^2, \alpha=0) \quad .$$

The difference $2(l-l_o)$ has under $H_o: \alpha = 0$ a χ^2 distribution with 1 degree of freedom. If we do not reject the hypothesis H_o this indicates the absence of competition effect and in this case we provide the analysis as in any block design.

3. SERIES OF EXPERIMENTS

It is now supposed that a series of experiments is carried out in J different environments (places or years). In each of this environments we compare the same set of treatments. However there is no assumption about the identity of designs, i.e. the experiments in different environments may be laid down in different block designs. As a result we have a vector of observations which may be represented as

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_J \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \dots & 0 \\ 0 & X_2 & \dots & 0 \\ & & & \\ 0 & 0 & \dots & X_J \end{bmatrix} \begin{bmatrix} \vartheta_1 \\ \vartheta_2 \\ \vartheta_J \end{bmatrix} + \begin{bmatrix} \alpha_1 W_1 & 0 & \dots & 0 \\ 0 & \alpha_2 W_2 & \dots & 0 \\ & & & \\ 0 & 0 & \dots & \alpha_J W_J \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_J \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_J \end{bmatrix} .$$

Using the formulae (1) and (2) we obtain

$$\hat{\Theta} = (\hat{\vartheta}_1, \hat{\vartheta}_2, \dots, \hat{\vartheta}_J) \quad ,$$

$$A = \mathrm{diag}(\hat{\sigma}_1^2 I, \hat{\sigma}_2^2 I, \dots, \hat{\sigma}_J^2 I)$$

and the conditional maximum likelihood function

$$L(Y|\Theta, A) = |2\pi\, G^{-1} A G^{-1}|^{(-1/2)} \exp[-(Y - G^{-1} X\hat{\Theta})' G A^{-1} G (Y - G^{-1} X\hat{\Theta})]$$

where

$$G = I - \mathrm{diag}(\alpha_1 W_1, \alpha_2 W_2, \dots, \alpha_J W_J)$$

and

$$X = \mathrm{diag}(X_1, X_2, \dots, X_J) .$$

The log-likelihood function has a form

$$\ln L = \sum_i \ln|G_i| - \sum_i (n_i/2) \ln(G_i y_i - X_i \hat{\vartheta}_i)'(G_i y_i - X_i \hat{\vartheta}_i) \quad . \tag{3}$$

This shows that the maximisation of $\ln L$ is equivalent to the maximisation of each $\ln L_i$ $(i=1,2,\dots,J)$.

Now the interesting point is whether all these competition coefficients are equal or not. To answer this question we have to find the maximum value of (3) but for a case in which all the G_i involve the same value α. The difference $2(l - l_o)$, where l is the maximum of (3) by different values of α and l_o is the maximum of (3) for a common α, has under the hypothesis $H_o: \alpha_1 = \alpha_2 = \dots = \alpha_J$ a χ^2 distribution with J-1 degrees of freedom. The rejection of H_o indicates that for the same set of varieties we have in different environments different competition coefficients. It seems that this may be caused by two facts. One is connected with the designs. Namely, if the designs were not equal and in the same time unbalanced in the sense of neigbourhood analysis, this may be reason for which H_o was rejected. However, if the designs were equal or balanced, the inequality of competition coefficients indicates the unequal behaviour of varieties in the considered environments what may be further investigated by the genotype-environments interaction analysis methods.

REFERENCES

Besag J.E., Kempton R.A. (1986). Statistical analysis of field experiments using neighbouring plots. *Biometrics* **42**, 231-251.

Dagnelie P. (1987). La methode de Papadakis en experimentation agronomique: considerations historiques et bibliographiques. *Biom. Praxim.* **27**, 49-64.

Dobek A., Kiełczewska H. (1987). Przegląd modeli doświadczeń polowych uwzględniających sąsiedztwo poletek. *Listy Biometryczne* **XXIV**, 2, 55-62.

Draper N.R., Guttman J. (1980). Incorporating overlap effects from neighbouring units into response surface models. *Appl. Statist.* **29**, 128-134.

Kempton R.A. (1982). Adjustment for competition between varieties in plant breeding trials. *J.Agric.Sci.* **98**, 599-611.

Mead R. (1967). A mathematical model for the estimation of inter-plant competition. *Biometrics* **23**, 189-205.

Pearce S.C. (1957). Experimenting with organisms as blocks. *Biometrika* **44**, 141-149.

Received 10 November 1990

PROPOZYCJA TESTU ISTOTNOSCI DLA WSPÓŁCZYNNIKA KONKURENCJI

Streszczenie

W pracy przedstawiono analizę doświadczenia, w którym występuje zjawisko konkurencji pomiędzy odmianami znajdującymi się na sąsiednich poletkach w bloku. Przedstawiono metodę testowania istotności współczynnika konkurencji w pojedynczym doświadczeniu oraz w serii doświadczeń.

Słowa kluczowe: konkurencja międzyodmianowa, metoda największej wiarogodności, seria doświadczeń